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Brief Communication

Scaling of the gas phase in particle-laden turbulent axisymmetric jets

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1. Introduction

Particle-laden (two-phase) turbulent axisymmetric jets have been the subject of research for more than 50 years because they have a large number of practical applications, particularly in the combustion of solid fuels. An understanding of the behaviour of particle-laden jets is necessary for the reliable modelling and optimisation of pulverised fuel burners because the interactions between the gas and the particles strongly influence heat transfer and pollutant emissions. Particle distributions are also important in many natural processes such as pollutant dispersion and the formation of rain. However, despite the significant advances in understanding, some basic scaling properties of particle-laden jets are yet to be reported. This task is addressed in this paper.

2. Results

It is well established (e.g. Hardalupas et al., 1989) that the addition of a solid phase to a turbulent axisymmetric jet alters its mean flow and turbulent structure when the exit mass loading, ϕ_o is significant. The exit mass loading is defined as

$$\phi_o = \frac{\dot{m}_p}{\dot{m}_f}, \tag{1}$$

where $\dot{m}_p$ is the solid phase mass flow rate and $\dot{m}_f$ is the gas phase exit mass flow rate. Previously published experimental work (e.g. Laats, 1966; Laats and Frishman, 1970; Elghobashi et al., 1984; Hardalupas et al., 1989) find that as ϕ_o increases, the centreline velocity decay rate, K_1, and the half width spreading rate, K_2 are both reduced.

The scaling of centreline velocity, u_c and half width, $r_{1/2}$ has been considered previously by Melville and Bray (1979). They began their analysis of particle-laden jets by writing the gas-phase velocity distribution as the following function of dimensionless parameters:

$$\frac{u}{u_o} = f_n\left(\frac{x}{D}, \frac{r}{D}, Re, \phi_o, St_o\right), \tag{2}$$

where u is the time-averaged axial component of the gas-phase velocity, u_o is the exit velocity of the gas phase, r is the radial distance from the jet axis, x is the axial distance from the nozzle exit and D is the nozzle diameter. The Reynolds number, $Re = \rho UD/\mu$, is defined using the bulk mean exit velocity U, the gas-phase density ρ and dynamic viscosity, μ. The Stokes number at the exit is defined as

$$St_o = \frac{\rho_p d_p^2 U}{18\mu D}, \tag{3}$$

where ρ_p is the particle density and d_p is the mean particle diameter. Initially, Melville and Bray neglect both Reynolds and Stokes numbers effects and they write the gas-phase centreline velocity as

$$\frac{u_o}{u_c} = K_1\left(\frac{x}{D}\right)a(\phi_o) \tag{4}$$

and the gas-phase velocity half width as

$$r_{1/2} = K_2(x)b(\phi_o), \tag{5}$$

where a and b are functions of ϕ_o.

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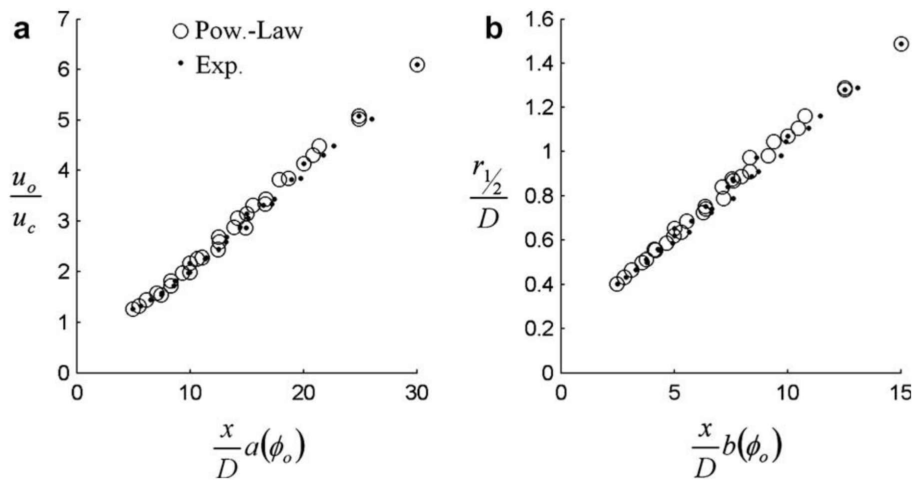

Fig. 1. Power-law and exponential correlations of (a) gas-phase centreline velocity and (b) half widths. The original measurements (St_o = 19) are reported in Laats (1966).

Table 1
The experimental conditions under which previous particle-laden pipe jet investigations were conducted where d_p is based mostly on the volume or mass averaged diameter except for Shuen et al. (1985), where d_p is the Sauter mean diameter

Author	Re	D (mm)	U (m/s)	d_p (μm)	ρ_p (kg/m^3)	St_o	ϕ_o
Budilarto (2003)	8400	14.2	8.9	25	2500	3	0.25–1.0
				70		24	
Gillandt et al. (2001)	5700	12	7.7	110	2000	48	1.0
Fan et al. (1997)	53,500	40	20	72	1250	10	0.22
Sheen et al. (1994)	20,000	15	20	210	1020	186	0.98, 2.75
				460		893	0.98, 3.06
				780		2568	0.49–3.06
Hardalupas et al. (1989)	13,000	15	13	80	2950	51	0.23, 0.86
Tsuji et al. (1988)	15,000	20	11	170	1020	50	2.0
	33,000		24	500		950	1.85
Hishida et al. (1985)	22,000	13	30	64	2590	76	0.3
Shuen et al. (1985)	22,000	10.9	30	79	2620	140	0.2
	19,000		25	119		264	0.2, 0.66
	19,000			207		799	0.66
Modarress et al. (1984a)	13,300	20	10	50	2990	12	0.32, 0.85
Modarress et al. (1984b)	14,100	20	10	200	2990	191	0.8
Subramanian and Raman (1984)	25,000	25.4	15	165	3200	160	0–2.5
Laats and Frishman (1970)	120,000	35	50	17	3950	5	0.3
				32		18	0.3–1.4
				49		42	0.3, 0.4
				72		91	0.3
Laats (1966)	74,000	27	40	40	2550	19	0.2–1.0

Measurements conducted prior to 1984, excluding those of Modarress et al. (1984a,b) used an isokinetic sampling tube and rotameter arrangement. Measurements conducted since 1984 used some LDA or PDA based technique.

Melville and Bray propose that a and b are exponential functions, giving

$$\frac{u_o}{u_c} = K_1\frac{x}{D}\,e^{-0.69\phi_o} \qquad (6)$$

and

$$\frac{r_{1/2}}{D} = K_2\frac{x}{D}\,e^{-0.69\phi_o}. \qquad (7)$$

With this, they were able to collapse the measurements reported in Laats (1966) as illustrated in Fig. 1 (dots). However, to our knowledge, nowhere is it reported that u_c and $r_{1/2}$ may alternately be scaled with power-law functions, where

$$\frac{u_o}{u_c} = K_1\frac{x}{D}\left(\frac{1}{1+\phi_o}\right) \qquad (8)$$

and

$$\frac{r_{1/2}}{D} = K_2\frac{x}{D}\left(\frac{1}{1+\phi_o}\right). \qquad (9)$$

The collapse of Laats' measurements in this way are also illustrated in Fig. 1 (circles). Since Melville and Bray (1979), numerous other experimental measurements of particle-laden jets have been reported (see Table 1). An inspection of these measurements reveals that power-law equations (8) and (9) collapse data for $St_o \lesssim 20$ (i.e. Laats, 1966; Laats and Frishman, 1970; Modarress et al., 1984a). For data of $St_o \gtrsim 20$, we have identified a further two different power-law scaling regimes. One regime covers the range $20 \lesssim St_o \lesssim 200$ and the other, $St_o \gtrsim 200$.

For $20 \lesssim St_o \lesssim 200$ (e.g. Tsuji et al., 1988; Hishida et al., 1985), we find that

$$\frac{u_o}{u_c} = K_1\frac{x}{D}\left(\frac{1}{1+\phi_o}\right) \qquad (10)$$

and

$$\frac{r_{1/2}}{D} = K_2\frac{x}{D}\left(\frac{1}{1+\phi_o}\right)^{1/2}. \qquad (11)$$

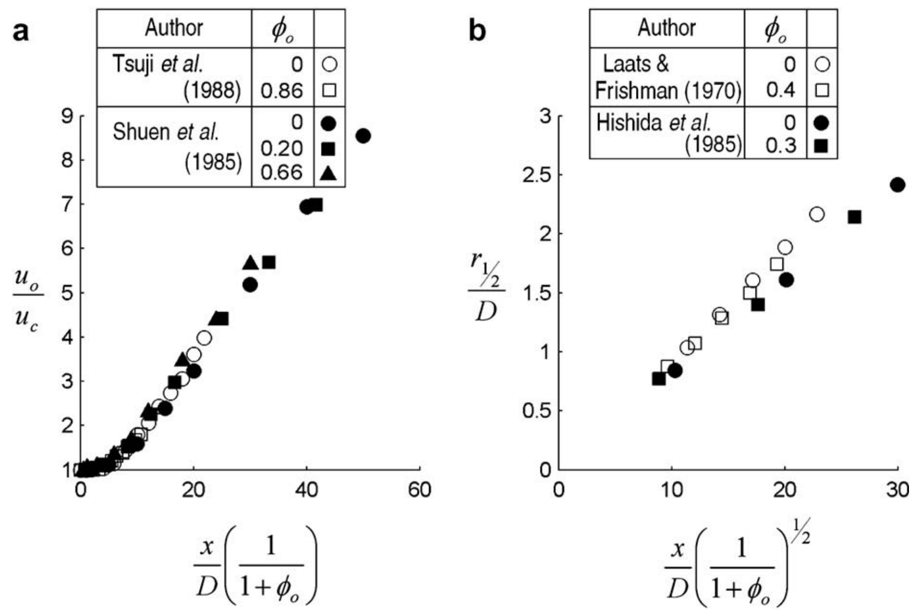

Fig. 2. (a) Gas-phase centreline velocity as reported by Tsuji et al. (1988) (open symbols, $St_o = 50$) and Shuen et al. (1985) (closed symbols, $St_o = 264$), but plotted as functions of $x/D(1+\phi_o)^{-1}$ and (b) gas-phase velocity half widths as reported by Laats and Frishman (1970) (open symbols, $St_o = 42$) and Hishida et al. (1985) (closed symbols, $St_o = 76$), but plotted as functions of $x/D(1+\phi_o)^{-1/2}$.

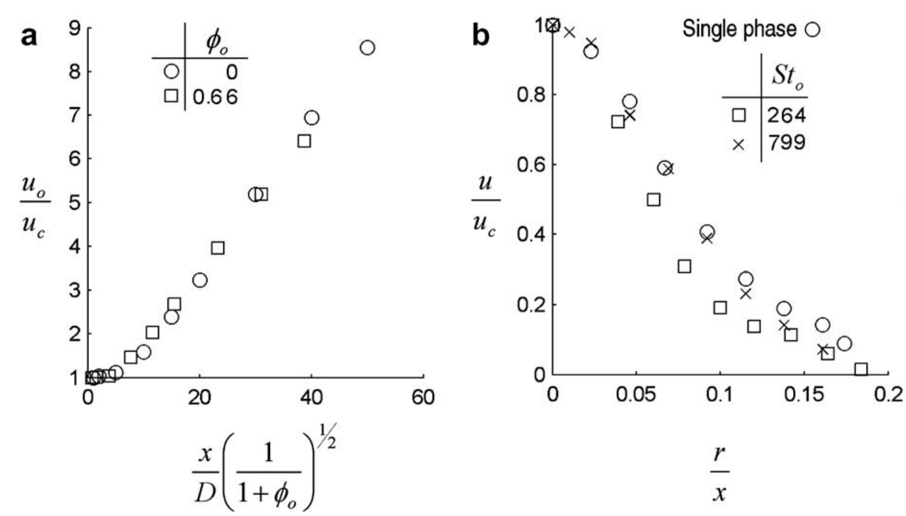

Fig. 3. Gas-phase centreline velocity as reported by Shuen et al. (1983), $St_o = 799$, but plotted as a function of $x/D(1+\phi_o)^{-1/2}$ and (b) radial velocity profiles ($x/D = 40$) as reported by Shuen et al. (1983), $\phi_o = 0.66$.

The result of scaling in this regime is illustrated in Fig. 2, where centreline velocity data (Tsuji et al., 1988; Shuen et al., 1985) and half widths (Laats and Frishman, 1970; Hishida et al., 1985) are shown to scale according to Eqs. (10) and (11), respectively. It is also evident that the slopes of scaled half widths in Fig. 2(b) are different between the measurements of Laats and Frishman (1970) and Hishida et al. (1985). This may be due to different boundary conditions in different experiments.

For $St_o \gtrsim 200$ (e.g. Sheen et al., 1994; Shuen et al., 1985), we find that

$$\frac{u_o}{u_c} = K_1 \frac{x}{D}\left(\frac{1}{1+\phi_o}\right)^{1/2} \tag{12}$$

and

$$\frac{r_{1/2}}{D} \approx K_2 \frac{x}{D}, \tag{13}$$

implying that half widths at large Stokes numbers are approximately independent of ϕ_o. Although neither Sheen et al. (1994) or Shuen et al. (1985) explicitly report half widths, an estimate of the dependence of jet spread on ϕ_o can be obtained by an inspection of their radial profiles. For example, shown in Fig. 3(b) are radial profiles at $x/D = 40$ for constant $\phi_o = 0.66$. It is evident that the profile corresponding to the case $St_o = 799$ closely matches the single phase case, while the profile of the case $St_o = 264$ is noticeably narrower.

The power-law scaling of the gas-phase centreline velocity and half-width for three regimes of exit Stokes number are summarised in Fig. 4. The strength of each correlation of measurements is assessed by the correlation coefficient defined as

$$R_{xu} = \frac{\sum(x_i-\overline{x})(u_i-\overline{u})}{\sqrt{\sum(x_i-\overline{x})^2\sum(u_i-\overline{u})^2}}, \tag{14}$$

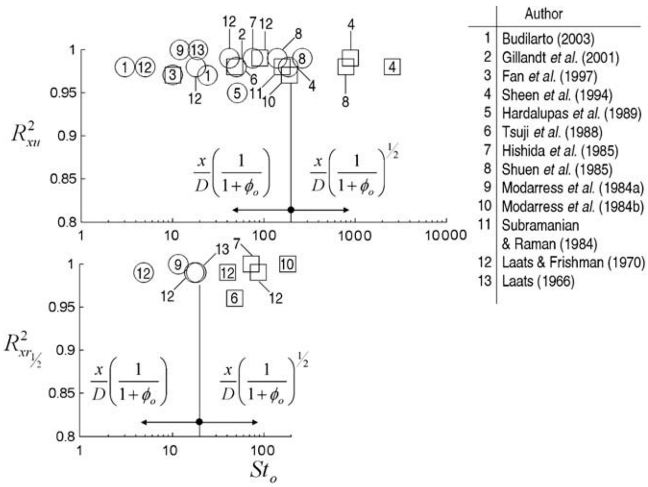

Fig. 4. Results of the linear correlation analysis of scaled gas-phase centreline velocities and half widths using previously published data. Circles and squares denote data that scaled with $x/D(1+\phi_o)^{-1}$ and $x/D(1+\phi_o)^{-1/2}$, respectively. Vertical lines indicate the approximate critical values.

where x_i represents $(x/D)(1+\phi_o)^{-n}$. Here, $n = 1$ or 1/2 (depending on St_o), u_i represents u_o/u_c values and the over bar denotes the mean. The half width correlation coefficient, $R_{xr_{1/2}}$ is defined similarly. Data points outside the region of linear jet growth ($x/D \lesssim 7$) are not included in the correlations. The squared correlation coefficient R^2_{xu} is used to accentuate departures from $R_{xu} \approx 1$. The correlations based on these power-law scalings are generally good. For example, the collapse of Laats' centreline and half width in Fig. 1(a) and (b) (power-law), yields $R^2_{xu} = 0.996$ and $R^2_{xr_{1/2}} = 0.994$, respectively. Likewise, scaling by Melville and Bray's factors yield $R^2_{xu} = 0.994$ and $R^2_{xr_{1/2}} = 0.992$. However, the advantages of the power-law scaling is it is applicable (where $n = 0$, 1/2 or 1) for the full range of St_o and gives a slightly better correlation (e.g. see above, Laats (1966) and Laats and Frishman (1970), $St_o = 18$, where $R^2_{xu} = 0.978$ and 0.964 for power-law and exponential scaling, respectively). Moreover, the power-law seems to have a physical basis, especially in the regime $St_o \gtrsim 200$, which corresponds to the scaling of a variable density jet (see Chen and Rodi, 1980). In this case, the mass of the solid phase is responsible for the greater jet density. For reference, Laats' unscaled data yields $R^2_{xu} = 0.69$ and $R^2_{xr_{1/2}} = 0.54$. See Table 1 for details of the experimental conditions for data used in Fig. 4.

Not included in Fig. 4 are measurements that are judged to have not been carried out far enough downstream to provide a reliable or strong linear correlation [Mostafa et al. (1989), Tsuji et al. (1988) for the cases other than $St_o = 50$] and flows issuing from smooth contractions (Ferrand et al., 2001). Circles and squares denote data that has been scaled with $x/D(1+\phi_o)^{-1}$ and $x/D(1+\phi_o)^{-1/2}$, respectively. Note that a circle overlaid with a square indicate Fan et al.'s (1997) measurements which scale with both $x/D(1+\phi_o)^{-1}$ ($\phi_o = 0.22$) and $x/D(1+\phi_o)^{-1/2}$ ($\phi_o = 0.8$) depending on the loading for $St_o = 10$.

The effectiveness of the scaling as illustrated in Fig. 4 was used to estimate the critical values of St_o. For $St_o \lesssim 200$, u_o/u_c scales with $x/D(1+\phi_o)^{-1}$, and within that range, $r_{1/2}$ scales with $x/D(1+\phi_o)^{-1/2}$ for $20 \lesssim St_o \lesssim 200$ and with $x/D(1+\phi_o)^{-1}$ for $St_o \lesssim 20$. For $St_o \gtrsim 200$, u_o/u_c scales with $x/D(1+\phi_o)^{-1/2}$. However, the critical value of 200 is of order of magnitude accuracy since some centreline velocity data scales with $x/D(1+\phi_o)^{-1/2}$ for $St_o \lesssim 200$ (Gillandt et al., 2001), and some with $x/D(1+\phi_o)^{-1}$ for $St_o > 200$ (Shuen et al., 1985). the particle size distribution is rarely fully monodisperse so that a range of St_o is present in each flow. This is expected to contribute to variation in the values of the regime boundaries along with the complicating effects of gravity and differences in boundary conditions between different experiments. These issues, as well as the sensitivity of R^2_{xu} to different exponents, are discussed further in Foreman (2008). Briefly, R^2_{xu} is sensitive (i.e. reduced) to a change in n, for data where ϕ_o has been varied significantly (e.g. Sheen et al., 1994, $\phi_o = 0 - 2.75$, $St_o = 186$), while the opposite is true for small parametric changes in ϕ_o (e.g. Hishida et al., 1985, $\phi_o = 0 - 0.3$). In the latter, the exact value of n can be ambiguous.

Some anomalous results include those of Modarress et al. (1984b), whose measurements of $r_{1/2}$ exhibit a clear dependence on ϕ_o, suggesting $r_{1/2}$ may scale according to Eq. (11), while the centreline scales with $x/D(1+\phi_o)^{-1/2}$. This may be related to that fact that the Stokes number of this data ($St_o = 191$) is very close to the proposed regime boundary (see Fig. 4), i.e. $St_o \approx 200$. Also, the gas-phase centreline velocities as reported by Fan et al. (1997) contradict the assumption that the scaling regime is a function of St_o only. Their centreline velocity for the case $\phi_o = 0.22$ is found to scale with $x/D(1+\phi_o)^{-1}$ while the case $\phi_o = 0.8$ scales with $x/D(1+\phi_o)^{-1/2}$, for a constant $St_o = 10$.

3. Conclusion

Exponential scaling factors that account for the first order influence of exit mass loading on the gas-phase centreline velocity and half width of a particle-laden jet were originally proposed by Melville and Bray (1979). However, the bulk of experimental data published since then suggests that the centreline velocity and half widths scale with a power-law factor.

Three regimes have been identified from the data presented in Table 1:

- For low Stokes numbers $St_o \lesssim 20$, the gas-phase centreline velocity and half width scale with $x/D(1+\phi_o)^{-1}$.
- For intermediate Stokes numbers, $20 \lesssim St_o \lesssim 200$, the gas-phase centreline velocity scales with $x/D(1+\phi_o)^{-1}$, and half width with $x/D(1+\phi_o)^{-1/2}$.
- For higher Stokes numbers, $St_o \gtrsim 200$, the gas-phase centreline velocity scales with $x/D(1+\phi_o)^{-1/2}$, while the half widths seem to be approximately independent of ϕ_o.

It is anticipated that the critical values separating these regimes will depend on the boundary conditions.

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